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A Decision Making Approach Based on Hesitant Fuzzy Linguistic-Valued Credibility Reasoning

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Abstract: Credibility reasoning has attracted a lot of attention due to its distinguished power and efficiency in representing uncertainty and vagueness within the process of reasoning and decision making. Aiming at the problem of inaccurate credibility estimation in uncertainty reasoning and making experts to express hesitant preferences better in evaluation reasoning process, this paper introduces hesitant fuzzy linguistic term set into credibility uncertainty reasoning. First, we propose hesitant fuzzy linguistic-valued credibility (HLCF), and establish the knowledge representation model of the hesitant fuzzy linguistic-valued credibility. Then, in order to solve the problem of incomplete information in the evaluation reasoning process, an information complement algorithm based on maximum similarity is constructed. After that, the algorithms of single rule and multiple rules of parallel relationship of hesitant fuzzy linguistic-valued credibility are proposed to enrich the reasoning rule base and get more accurate reasoning results. The closeness degrees between the conclusions of each alternative after reasoning and the expected value are calculated, so as to select the most suitable alternative. Finally, a practical example which concerned the social risk analysis is given to illustrate the applicability and effectiveness of the proposed approach.

Key Words: Hesitant fuzzy linguistic-valued credibility, Maximum similarity, Uncertainty reasoning, Knowledge representation
Categories: M.1, M.5
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1 Introduction

Knowledge reasoning is an important research direction in the field of artificial intelligence. In the reasoning process, according to whether the knowledge is deterministic, it can be divided into certainty reasoning and uncertainty reasoning. Both traditional inductive reasoning and deductive reasoning belong to certainty reasoning, indicating the precise concept of “one or the other”. However, the randomness and uncertainty of objective things or phenomena in the real world lead to the fact that most of the knowledge in various cognitive fields is uncertain, so a lot of knowledge has the properties of uncertainty, vagueness and randomness. The study in the field of artificial intelligence is particularly important.

Since the emergence of the first expert system in [Liu et al. 1967], various uncertainty studies have attracted extensive attention from experts and scholars in various fields. How to deal with and express uncertainty knowledge has become one of the most important topics in artificial intelligence research. Among them, the credibility-based method [Gao et al. 2019] [Gottifredi et al. 2018] [Snow, 2017] introduces credibility into the knowledge or phenomenon of uncertainty, which makes the original fuzzy knowledge or phenomenon become quantitative and clear, and has received extensive attention and research from experts at domestic and abroad. [Shortiliffe, 1976] combined with probability theory to propose a reasoning method based on credibility; [Qian et al. 2009] solved the problem of uncertain information by using rule-based credibility reasoning; [Wu, 1993] made a credibility estimate for the resolution of the basic clauses in fuzzy logic; [Chen et al. 2005] established two reasoning mechanisms with a single conclusion, multiple conditions and credibility in both environments where the conditions are precise and fuzzy; [Xiong et al. 2014] established a credibility debate model, which can effectively deal with the process of debate reasoning under the condition of uncertain information.

On the other hand, [Zadeh, 1975] proposed the fuzzy set theory, which extends the research scope of various fields in modern society from precision to fuzzification. This method takes the objects and the fuzzy concepts reflecting them as a fuzzy set, then introduces the concept of membership degree, and establishes the appropriate membership function to describe the fuzziness of the elements in the fuzzy set. On this basis, [Torra and Narukawa, 2009] [Torra, 2010] proposed a new extended fuzzy set form, namely hesitant fuzzy set, which is used to manage the hesitation of experts among several values. The membership degree of a hesitant fuzzy set is no longer a certain value, but a set of possible values, which is closer to real life and has a unique advantage in dealing with uncertainty reasoning, that is, to avoid the loss of information caused by the aggregation operators as much as possible. Hesitant fuzzy set has received extensive attention and research from many scholars [Ranjbar et al. 2018] [Quirós et al. 2017] [Liu et al. 2017]. [Xu et al. 2013] proposed distance and similarity measures for hesitant fuzzy sets based on numerical values. [Xia et al. 2013] also proposed some hesitant fuzzy aggregation operators and applied them to group decision making. [Wei, 2012] constructed some priori aggregation operators for hesitant fuzzy information, and developed some models for solving hesitant fuzzy multi-attribute decision problems, in which each attribute has different priorities.

It should be noted that the HFS was introduced to handle the problems which are represented in quantitative situations. However, uncertainty is produced by the vagueness of meanings whose nature is qualitative rather than quantitative in many cases [Chen et al. 2014] [Herrera et al. 2008] [Li et al. 2018a]. People are more accustomed to express with linguistic values rather than numerical values when conducting evaluation reasoning. For example, when we evaluate the ‘‘quality’’ of a teacher's teaching, the linguistic terms such as ‘‘good’’, ‘‘very good’’, ‘‘general’’ might be used. On this basis, motivated by HFSs and linguistic fuzzy sets, [Rodríguez et al. 2012] proposed the concept of the hesitant fuzzy linguistic term set (HFLTS). It is defined as a methodology for reasoning, computing and making decisions using information described in natural language [Mendel et al. 2010]. HFLTS can simulate the hesitancy of decision makers when elicits linguistic preferences. Therefore, it not only provides tools close to human beings reasoning processes related to decision making, which can emulate human cognitive processes better, but also enhances the reliability and flexibility of classical decision models and improves the resolution of decision making under uncertainty with linguistic information [Li et al. 2018b] [Zhang et al. 2018] [Rodríguez et al. 2012] [Rodríguez et al. 2016]. It permits decision makers to use several linguistic terms to assess a linguistic variable. Thus, it provides many advantages in depicting decision makers’ cognitions and preferences. [Liao et al. 2014] proposed various methods for calculating the distance and similarity measures between hesitant fuzzy linguistic terms, and applied linguistic information to multi-criteria decision-making problems. Subsequently, [Zhu et al. 2014] introduced hesitant fuzzy linguistic preference relation as a tool to acquire and represent experts’ preferences, and studied the consistency of the hesitant fuzzy linguistic preference relation.

There are some limitations in the credibility uncertainty reasoning, for example, the method requires that the chain of reasoning for solving the problem cannot be too long. If the chain of reasoning is longer, the reasoning error caused by the inaccurate estimation of the credibility will be more. And considering that people are often accustomed to using linguistic values for reasoning evaluation, the process of converting linguistic values into numerical values will result in a large amount of information missing. Therefore, this paper combines the credibility uncertainty reasoning with the hesitant fuzzy linguistic term set, on this basis, the concept of hesitant fuzzy linguistic-valued credibility is proposed, which minimizes the inaccuracy of credibility valuation and makes the reasoning result more accurate.

Based on this focus, the rest of this paper is organized as follows: Section [2] first presents the concepts of linguistic term set and hesitant fuzzy language term set. Then, we give the definition of the hesitant fuzzy linguistic-valued credibility, and propose the representation method for hesitant fuzzy linguistic knowledge and the hesitant fuzzy linguistic-valued credibility reasoning rules. In Section [3], to address the problem that information is incomplete, an information complementation algorithm based on maximum similarity is constructed. In Section [4], we mainly study the method of a single rule supporting conclusion and multiple rules of parallel relationship supporting conclusion in hesitant fuzzy linguistic-valued credibility reasoning. Giving the expected value of the conclusion, furthermore, the closeness degrees between the conclusions of each alternative after reasoning and the expected value is calculated. Therefore, we can select the best alternative. Section [5] uses a practical example involving social risk analysis to verify the efficiency and applicability of the proposed approach, and makes a compared analysis with the reference [Gao et al. 2019]. Finally, the paper finishes with some concluding and outlooks in Section [6].

2 Hesitant Fuzzy Linguistic-valued Credibility

2.1 Hesitant Fuzzy Linguistic Term Set

In this section we will review some necessary concepts, such as linguistic term set and hesitant fuzzy linguistic term set.

Definition 1 ([Herrera et al. 2000]) Let $S=\{s_\alpha \mid \alpha=-n,...,-1,0,1,...,n\}$ be a set of finite full-order linguistic terms, consisting of an odd number of linguistic terms, then *S* satisfies the following properties:

(1)Order, $s_i \le s_j \Leftrightarrow i \le j$;

(2)Reversibility, $\mathrm{Neg}(s_{-i})=s_i$, where $-i+i=0$;

(3)Boundedness, s_{-n} and s_n are the lower and upper bounds of linguistic labels where n is a positive integer.

where the mid linguistic label S_0 represents an assessment of ''indifference'', and the rest of them are placed symmetrically around it.

Definition 2 ([Liao et al. 2014]) Let $S=\{s_\alpha \mid \alpha=-n,...,-1,0,1,...,n\}$ be a set of finite full-order linguistic terms, consisting of an odd number of linguistic terms. The discrete linguistic term set *S* extends to the continuous linguistic term set $\overline{S}=\{s_\alpha \mid \alpha\in[-q,q]\}$, where q $(q>n)$ is a sufficiently large positive integer. In general, the linguistic term $s_\alpha(s_\alpha\in S)$ is given by the decision maker, while the extended linguistic term (also named virtual linguistic term) $\overline{s_\alpha}(\overline{s_\alpha}\in\overline{S})$ only appears in computation.

For any two linguistic terms $s_\alpha, s_\beta \in \overline{S}$ and $\lambda,\lambda_1,\lambda_2\in[0,1]$, the following operational laws were introduced:

(1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;

(2) $\lambda s_\alpha = s_{\lambda\alpha}$;

(3) $(\lambda_1+\lambda_2)s_\alpha=\lambda_1 s_\alpha \oplus \lambda_2 s_\alpha$;

(4) $\lambda(s_\alpha\oplus s_\beta)=\lambda s_\alpha \oplus \lambda s_\beta$.

Definition 3 ([Rodríguez et al. 2012]) Let $S=\{s_\alpha \mid \alpha=-n,..., -1,0,1,...,n\}$ be a linguistic term set, an HFLTS, H_S, is an ordered finite subset of the consecutive linguistic terms of *S*.

Let *S* be a linguistic term set, $S=\{s_\alpha \mid \alpha=-n,..., -1,0,1,...,n\}$, we then define the empty HFLTS and the full HFLTS for a linguistic variable ϑ as follows.

1) Empty HFLTS: $H_S(\vartheta)=\{\}$,

2) Full HFLTS: $H_S(\vartheta)=S$.

Any other HFLTS is formed with at least one linguistic term in *S* and $\boldsymbol{H_S}$ is the set of all HFLEs.

2.2 Knowledge Representation with Hesitant Fuzzy Linguistic-valued Credibility

In uncertainty knowledge representation based on credibility, as the values of credibility factor are different, the number of rules in the knowledge base increases which leads to the efficiency of reasoning also decreasing. And when the chain of reasoning is long, the reasoning error due to the inaccurate estimation of credibility will be more. On the other hand, human beings are more accustomed to using linguistic values for evaluation and reasoning in their daily lives. In order to solve these problems, this paper introduces hesitant fuzzy linguistic term set into credibility uncertainty knowledge representation and reasoning, which expands the credibility of rules, premises and conclusions into hesitant fuzzy linguistic term set composed of multiple linguistic-valued membership degrees.

The hesitant fuzzy linguistic-valued credibility (HLCF) is the truth degree on judging a thing or a phenomenon which given by experts who disagree with or cannot persuade with each other, including premise credibility and rule credibility.

Definition 4 Let $A = \{a_i \mid i = 1, 2, ..., m\}$, in the knowledge representation model of HLCF, hesitant fuzzy linguistic knowledge can be expressed as

$$(A, HLCF(A)), \tag{1}$$

where A represents the premise, and $HLCF(A)$ is the hesitant fuzzy linguistic-valued credibility of A, $HLCF(A) = \{LCF_S(a_1), LCF_S(a_2), ..., LCF_S(a_m) \mid a_i \in A, i = 1, 2, ..., m\}$, $LCF_S(a_i)$ is the element in $HLCF(A)$.

Remark 1.

1) $LCF_S(a_i) \in [s_{-n}, s_n]$.
2) For $\forall LCF_S(a_i) \in HLCF(A)$, $i = 1, 2, ..., m$, if $LCF_S(a_i) = s_n$, then A is definitely credibility; If $LCF_S(a_i) = s_{-n}$, then A is definitely incredibility; If $LCF_S(a_i) = s_0$, then it cannot judge whether A is credibility or not.
3) For $\forall LCF_S(a_i) \in HLCF(A)$, $i = 1, 2, ..., m$, when $s_{-n} < LCF_S(a_i) < s_0$, A is incredibility by the degree of $HLCF(A)$; Correspondingly, when $s_0 < LCF_S(a_i) < s_n$, A is credibility by the degree of $HLCF(A)$.

Definition 5 In the knowledge representation model of HLCF, the IF-THEN rules can be expressed as:

IF $(A(a_i), HLCF_S(a_i))$ THEN $(H(h_s), HLCF_S(h_i))$ $(HLCF(H(h_s), A(a_i)))$

in which, $A(a_i)$ is a combination of some logical relationships, $H(h_s)$ is the conclusion, $HLCF(H(h_s), A(a_i))$ is the hesitant fuzzy linguistic-valued credibility of the rule, and $LCF_S(h_s, a_i)$ is the element in $HLCF(H(h_s), A(a_i))$, which has a value range of $[s_{-n}, s_n]$.

Remark 2.

1) Give a threshold $s_\beta \in [s_{-n}, s_n]$, if $\forall LCF_S(h_s, a_i) > s_\beta$ in HLCF, then the existence of $A(a_i)$ increases the credibility of the establishment of $H(h_s)$; IF $\forall LCF_S(h_s, a_i) < s_\beta$, then the existence of $A(a_i)$ increases the credibility that $H(h_s)$ does not hold.
2) For $\forall LCF_S(h_s, a_i) \in HLCF(H(h_s), A(a_i))$, $i = 1, 2, ..., m$, if $LCF_S(h_s, a_i) = s_{-n}$, then the existence of $A(a_i)$ must make $H(h_s)$ not true; If $LCF_S(h_s, a_i) = s_n$, the existence of $A(a_i)$ must make $H(h_s)$ be true; If $LCF_S(h_s, a_i) = s_\beta$, then the existence of the premise has no effect on whether the conclusion is true or not.

3 Complete Algorithm Based on Maximum Similarity

In view of the fact that the knowledge reasoning process of the hesitant fuzzy linguistic-valued credibility elements will have insufficient knowledge and experience of the reasoner and improper data storage, which often causes the lack of evaluation information. To address this problem, this paper studies a complete algorithm based on maximum similarity of the incomplete hesitant fuzzy linguistic-valued credibility elements.

Definition 6 Let A be a non-empty finite set, and $HLCF(A_\phi)=\{LCF_S(a_i)\mid a_i\in A, i=1,2,...,m, \phi=1,2,...,l\}$ be a hesitant fuzzy linguistic-valued credibility element (HLCFE(s)) on A. If $m_i^a\neq(m_i^a)^*$, then the HLCFE(s) are called incomplete hesitant fuzzy linguistic-valued credibility elements, denoted by IHLCFE(s); The matrix(s) composed of IHLCFE(s) are called incomplete hesitant fuzzy linguistic-valued credibility matrix(s), denoted by IHLCFM(s); The set(s) composed of IHLCFE(s) are called incomplete hesitant fuzzy linguistic-valued credibility set(s), denoted by IHLCFS(s), where m_i^a is the number of values in $HLCF(A_\phi)$, $(m_i^a)^*$ is the largest value of m_i^a.

Definition 7 Let $HLCF(A_1)=\{LCF_S(a_i)\mid a_i\in A_1, i=1,2,...,m\}$ and $HLCF(A_2)=\{LCF_S(a_j)\mid a_j\in A_2, j=1,2,...,m\}$ be two HLCFEs, the similarity degree $sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))$ between $HLCF(A_1)$ and $HLCF(A_2)$ is defined as:

$$sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))=\frac{\sum_{i,j=1}^{m}LCF_S(a_i)\,.\,LCF_S(a_j)}{\sqrt{\sum_{i=1}^{m}LCF_S(a_i)^2}\,.\,\sqrt{\sum_{j=1}^{m}LCF_S(a_j)^2}} \tag{2}$$

Example 1 Let $U=\{A_1,A_2\}$ is a non-empty set, and assume $HLCF(A_1)=\{s_{-2},s_0,s_2\}$ and $HLCF(A_2)=\{s_1,s_3,s_4\}$ be two HLCFEs, calculating the similarity degree $sim_{HLCFEs}(HLCF(A_1), HLCF(A_2))$ between $HLCF(A_1)$ and $HLCF(A_2)$. According to the equation (2),

$$sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))=\frac{s_{-2}\times s_1+s_0\times s_3+s_2\times s_4}{\sqrt{s_{(-2)^2}+s_{0^2}+s_{2^2}}\times\sqrt{s_{1^2}+s_{3^2}+s_{4^2}}}=s_{0.416}$$

Theorem 1 Let $HLCF(A_1)$, $HLCF(A_2)$ and $HLCF(A_3)$ be any three HLCFEs, then the similarity degree between any two of them has the following properties:

(1) $s_{-1}\le sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))\le s_1$;

(2) $sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))=s_1$, if and only if $HLCF(A_1)=HLCF(A_2)$

(3) $sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))=sim_{HLCFEs}(HLCF(A_2),HLCF(A_1))$;

(4) If $HLCF(A_1)\subseteq HLCF(A_2)\subseteq HLCF(A_3)$, then $sim_{HLCFEs}(HLCF(A_1),HLCF(A_2))\ge sim_{HLCFEs}(HLCF(A_1),HLCF(A_3))$, $sim_{HLCFEs}(HLCF(A_2),HLCF(A_3))\ge sim_{HLCFEs}(HLCF(A_1),HLCF(A_3))$.

Proof.

(1) According to the equation (2), since $s_{-n} \le LCF_S(a_i) \le s_n$ and $s_{-n} \le LCF_S(a_j) \le s_n$, then we have $s_{-n^2m} \le \sum_{i,j=1}^{m} LCF_S(a_i) \,.\, LCF_S(a_j) \le s_{n^2m}$, $\sqrt{\sum_{i=1}^{m} LCF_S(a_i)^2} \le s_{n.\sqrt{m}}$ and $\sqrt{\sum_{j=1}^{m} LCF_S(a_j)^2} \le s_{n.\sqrt{m}}$, so we can get $\sqrt{\sum_{i=1}^{m} LCF_S(a_i)^2} \,.\, \sqrt{\sum_{j=1}^{m} LCF_S(a_j)^2} \le s_{n^2m}$. Thus, the equation (2) satisfies

$$s_{-1} \le \frac{\sum_{i,j=1}^{m} LCF_S(a_i) \,.\, LCF_S(a_j)}{\sqrt{\sum_{i=1}^{m} LCF_S(a_i)^2} \,.\, \sqrt{\sum_{j=1}^{m} LCF_S(a_j)^2}} \le s_1.$$

(2) According to definition 4 and definition 7, the (2), (3) and (4) of the Theorem 1 can be directly verified.

Definition 8 Let $HLCF(A_1) = \{LCF_S(a_i) \mid a_i \in A_1, i = 1,2,...,m\}$ and $HLCF(A_2) = \{LCF_S(a_j)) \mid a_j \in A_2, j = 1,2,...,m\}$ be two HLCFEs, then the similarity matrix of $HLCF(A_1)$ and $HLCF(A_2)$ on attribute $c_r (1 \le r \le q)$ is defined as:

$$R_{c_r} = (x_{ij})_{m\times m}, \text{ where } x_{ij} = sim_{HLCFEs}^{c_r}(HLCF(A_1), HLCF(A_2)) \tag{3}$$

According to the definition 8, we can get the following properties easily:

Theorem 2 Let $HLCF(A_1) = \{LCF_S(a_i) \mid a_i \in A_1, i = 1,2,...,m\}$ and $HLCF(A_2) = \{LCF_S(a_j) \mid a_j \in A_2, j = 1,2,...,m\}$ be two HLCFEs, $\forall x_{ij} \in R_{c_r}$, then the similarity matrix $R_{c_r} = (x_{ij})_{m\times m}$, where $1 \le r \le q$ satisfies the following properties:

(1) $x_{ij} \in [s_{-1}, s_1]$;

(2) If $i = j$, then $x_{ii} = s_1$;

(3) R_{c_r} is a symmetric matrix.

Definition 9 Let $HLCF(A_1) = \{LCF_S(a_i) \mid a_i \in A_1, i = 1,2,...,m\}$ and $HLCF(A_2) = \{LCF_S(a_j) \mid a_j \in A_2, j = 1,2,...,m\}$ be two HLCFEs, q is the number of attributes, then the similarity aggregation matrix R_η is defined as:

$$R_\eta = (y_{ij})_{m\times m} \tag{4}$$

where

$$y_{ij} = sim_\eta(HLCF(A_1), HLCF(A_2)) = \frac{\sum_{r=1}^{q} sim_{HLCFEs}^{c_r}(HLCF(A_1), HLCF(A_2))}{q} \tag{5}$$

Remark 3 $sim_\eta(HLCF(A_1), HLCF(A_2))$ is denoted by the similarity aggregation degree of $HLCF(A_1)$ and $HLCF(A_2)$ on q attributes, which satisfies all the properties in Theorem 1.

According to the definition 9, we can get the following properties easily:

Theorem 3 Let $HLCF(A_1) = \{LCF_S(a_i) \mid a_i \in A_1, i = 1,2,...,m\}$ and $HLCF(A_2) = \{LCF_S(a_j) \mid a_j \in A_2, j = 1,2,...,m\}$ be two HLCFEs, $\forall y_{ij} \in R_\eta$, then the similarity aggregation matrix R_η satisfies the following properties:

(1) $y_{ij} \in [s_{-1}, s_1]$;

(2) If $i = j$, then $y_{ii} = s_1$;

(3) R_η is a symmetric matrix.

Definition 10 Let $HLCF(A_\gamma) = \{LCF_S(a_i) \mid a_i \in A_\gamma, i = 1,2,...,m\}$ be any IHLCFE(s), and the two of them are:

$$HLCF(A_1) = \{LCF_S(a_{11}), LCF_S(a_{12}), ..., LCF_S(a_{1v})\}$$
$$HLCF(A_2) = \{LCF_S(a_{21}), LCF_S(a_{22}), ..., LCF_S(a_{2w})\}$$

Let $(m_i^a)^*$ be the maximum number of elements in $HLCF(A_\gamma)$. Calculate the similarity of any two HLCFEs after initial completion according to equation (2), if $HLCF(A_1)$ and $HLCF(A_2)$ have the maximum similarity $sim_{\max}$, the complete formula based on the maximum similarity are defined as follows:

$$\delta(\Delta_{ij}) = \begin{cases} \dfrac{LCF_S(a_{21}) + LCF_S(a_{22}) + ... + LCF_S(a_{2w})}{w} * sim, & v < w = (m_i^a)^* \\ \dfrac{LCF_S(a_{11}) + LCF_S(a_{12}) + ... + LCF_S(a_{1v})}{v} * sim, & w < v = (m_i^a)^* \end{cases} \tag{6}$$

Remark 4.

1) Δ_{ij} indicates the missing values of the j_{th} attribute with the i_{th} alternative in IHLCFEs, if $v < w = (m_i^a)^*$, then add the $(m_i^a)^* - v$ number of Δ_{ij} in $HLCF(A_1)$.

2) If both v and w are less than $(m_i^a)^*$, the larger one is used to complement the smaller one. And if $v=w < (m_i^a)^*$, the two of them complete with each other.

4 Uncertainty Reasoning Based on Hesitant Fuzzy Linguistic-valued Credibility

In daily life, most of the information is complex, fuzzy and incomplete. So, it is necessary to solve the problem of uncertainty information. We study the uncertainty reasoning based on hesitant fuzzy linguistic-valued credibility.

For example, the following is a rule in the medical evaluation system:

IF The doctor has sufficient medical knowledge and rich clinical experience.

THEN The doctor can be a very good surgeon.

This rule is a heuristic rule. Assuming premise P is that the doctor has sufficient medical knowledge, Q is that the doctor has rich clinical experience, and conclusion H is that the doctor can be a very good surgeon, then there is a rule $P(0.7) \wedge Q(0.8) \rightarrow H(0.7)$, i.e. if $P(0.7) \wedge Q(0.8)$ is true, then the probability that true of H is 70%.

Obviously, "sufficient" in the premise P and "rich" in the premise Q are fuzzy concepts that are evaluated by linguistic values. In the process of reasoning, the conversion of linguistic values into numerical values will cause a large amount of missing information, and due to the evaluation values of different experts influence the results of the rules, resulting in inaccurate values of the aggregation process. Therefore, a set of hesitant linguistic values can be used to indicate the degree of premises. The degree of premise is different, and the credibility of the conclusion is also different. In the same way, the credibility of the rule establishment can also be represented by a set of hesitant linguistic values.

4.1 Hesitant Fuzzy Linguistic-valued Credibility and Its Reasoning Method

Definition 11 Let $HLCF(A)=\{LCF_S(a_1),LCF_S(a_2),...,LCF_S(a_n)\}$, $HLCF(B)=\{LCF_S(b_1),LCF_S(b_2),...,LCF_S(b_n)\}$, the commonly used operators in the HLCF are defined as follows:

1) $HLCF(A)\vee HLCF(B)=\{LCF_S(a_1),LCF_S(a_2),...,LCF_S(a_n)\}\vee\{LCF_S(b_1), LCF_S(b_2),...,LCF_S(b_n)\}=\{LCF_S(a_1)\vee LCF_S(b_1),LCF_S(a_2)\vee LCF_S(b_2),...,LCF_S(a_n)\vee LCF_S(b_n)\}$;

2) $HLCF(A)\wedge HLCF(B)=\{LCF_S(a_1),LCF_S(a_2),...,LCF_S(a_n)\}\wedge\{LCF_S(b_1), LCF_S(b_2),...,LCF_S(b_n)\}=\{LCF_S(a_1)\wedge LCF_S(b_1),LCF_S(a_2)\wedge LCF_S(b_2),...,LCF_S(a_n)\wedge LCF_S(b_n)\}$;

3) $HLCF(A)'=1-HLCF(A)=\{1-LCF_S(a_1),1-LCF_S(a_2),...,1-LCF_S(a_n)\}$.

The reasoning rule of hesitant fuzzy linguistic-valued credibility has two cases: single rule support conclusion and multiple rules which have parallel relationship support conclusion.

Definition 12 If a single rule supports the conclusion, let A be a hesitant fuzzy linguistic term set on the state space (set), which is the credibility of the rule's predecessor, and H is the conclusion of a proposition, $LCF_S(h_i,a_i)$ is the element of $HLCF(H(h_i),A(a_i))$, $LCF_S(h_i,a_i)\in[s_{-n},s_n]$($i$=1,2,…,m), then $HLCF(\ H(h_i),A(a_i))$ is the hesitant fuzzy linguistic-valued credibility of the reasoning rule

$$A\to H \qquad HLCF(H(h_i),A(a_i)) \tag{7}$$

Where the object set $A=\{a_1,a_2,...,a_m\}$, let $HLCF_S(a_i)=T$, according to $A\to H \quad HLCF(H(h_i),\ A(a_i))$, we can know that the linguistic-valued credibility of H on object A is $HLCF_S(h_i)$. It not only depends on hesitant fuzzy linguistic-valued membership degree $HLCF_S(a_i)$, but also relies on the hesitant fuzzy linguistic-valued credibility $HLCF(H(h_i),A(a_i))$ of the rule, that is, $HLCF_S(h_i)$ is a function of $HLCF_S(a_i)$ and $HLCF(H(h_i),A(a_i))$, which is denoted by ***R***, e.t. $HLCF_S(h_i)=R(T,HLCF(H(h_i),A(a_i)))$. ***R*** is the function of $[s_{-n},s_n]\times[s_{-n},s_n]\to[s_{-n},s_n]$ called ***R*** operator, and its reasoning model is as follows

$$A \qquad \to \qquad H \qquad\qquad HLCF(H(h_i),A(a_i))$$

$HLCF_S(a_i) = T$	
	$HLCF_S(h_i) = R(T, HLCF(H(h_i), A(a_i)))$

where the calculation method of the ***R*** operator is

$$HLCF_S(h_i) = \boldsymbol{R}(T, HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T \times HLCF(H(h_i), A(a_i)) \tag{8}$$

Theorem 4 Let $T' = \{s_n, s_n, ..., s_n\}$, $T'' = \{s_0, s_0, ..., s_0\}$, the ***R*** operator satisfies the following properties:

1) Unity: $\boldsymbol{R}(T', HLCF(H(h_i), A(a_i))) = HLCF(H(h_i), A(a_i))$, $\boldsymbol{R}(T, T') = T$;
2) Zero elementality: $\boldsymbol{R}(T'', HLCF(H(h_i), A(a_i))) = \boldsymbol{R}(T, T'') = T''$;
3) Monotonicity: If $T_1 \subseteq T_2 \subseteq HLCF(H(h_i), A(a_i))$, then $\boldsymbol{R}(T_1, HLCF(H(h_i), A(a_i))) \subseteq \boldsymbol{R}(T_2, HLCF(H(h_i), A(a_i)))$;
   If $HLCF(H(h_i), A(a_i)) \subseteq HLCF(H(h_i), B(b_i)) \subseteq T$, then $\boldsymbol{R}(T, HLCF(H(h_i), A(a_i))) \subseteq \boldsymbol{R}(T, HLCF(H(h_i), B(b_i)))$.

Proof.

1) Let $HLCF(H(h_i), A(a_i)) = \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\}$, $T = \{LCF_S(a_1), LCF_S(a_2), ..., LCF_S(a_m)\}$.

Known by definition 2 and definition 12, $\boldsymbol{R}(T', HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T' \times HLCF(H(h_i), A(a_i)) = \frac{1}{n} \times \{s_n, s_n, ..., s_n\} \times \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\} = \{\frac{1}{n} \times s_n \times LCF_S(h_1, a_1), \frac{1}{n} \times s_n \times LCF_S(h_2, a_2), ..., \frac{1}{n} \times s_n \times LCF_S(h_m, a_m)\} = \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\} = HLCF(H(h_i), A(a_i))$.

In the same way, we can get $\boldsymbol{R}(T, T') = \frac{1}{n} \times T \times T' = \frac{1}{n} \times \{LCF_S(a_1), LCF_S(a_2), ..., LCF_S(a_m))\} \times \{s_n, s_n, ..., s_n\} = \{\frac{1}{n} \times LCF_S(a_1) \times s_n, \frac{1}{n} \times LCF_S(a_2) \times s_n, ..., \frac{1}{n} \times LCF_S(a_m) \times s_n\} = \{LCF_S(a_1)), LCF_S(a_2), ..., LCF_S(a_m)\} = T$.

In summary, the ***R*** operator satisfies unity.

2) Let $HLCF(H(h_i), A(a_i)) = \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\}$, $T = \{LCF_S(a_1), LCF_S(a_2), ..., LCF_S(a_m)\}$.

According to definition 2 and definition 12, $\boldsymbol{R}(T'', HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T'' \times HLCF(H(h_i), A(a_i)) = \frac{1}{n} \times \{s_0, s_0, ..., s_0\} \times \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\} = \{\frac{1}{n} \times s_0 \times LCF_S(h_1, a_1), \frac{1}{n} \times s_0 \times LCF_S(h_2, a_2), ..., \frac{1}{n} \times s_0 \times LCF_S(h_m, a_m)\} = \{s_0, s_0, ..., s_0\} = T''$.

Similarly, we can prove $\boldsymbol{R}(T, T'') = T''$.

To sum up, the ***R*** operator satisfies the zero elementality.

3) Let $HLCF(H(h_i), A(a_i)) = \{LCF_S(h_1, a_1), LCF_S(h_2, a_2), ..., LCF_S(h_m, a_m)\}$,

$T_1 = \{LCF_S^1(a_1), LCF_S^1(a_2), ..., LCF_S^1(a_m)\}$, $T_2 = \{LCF_S^2(a_1), LCF_S^2(a_2), ..., LCF_S^2(a_n)\}$.

From definition 2 and definition 12, we can know that $\boldsymbol{R}(T_1, HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T_1 \times HLCF(H(h_i), A(a_i))$, $\boldsymbol{R}(T_2, HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T_2 \times HLCF(H(h_i), A(a_i))$, because $T_1 \subseteq T_2$, we can get $\frac{1}{n} \times T_1 \times HLCF(H(h_i), A(a_i)) \le \frac{1}{n} \times T_2 \times HLCF(H(h_i), A(a_i))$, that is $\boldsymbol{R}(T_1, HLCF(H(h_i), A(a_i))) \subseteq \boldsymbol{R}(T_2, HLCF(H(h_i), A(a_i)))$.

Similarly, $\boldsymbol{R}(T, HLCF(H(h_i), A(a_i))) = \frac{1}{n} \times T \times HLCF(H(h_i), A(a_i))$, $\boldsymbol{R}(T, HLCF(H(h_i), B(b_i))) = \frac{1}{n} \times T \times HLCF(H(h_i), B(b_i))$, when $HLCF(H(h_i), A(a_i)) \subseteq HLCF(H(h_i), B(b_i))$, we can get $\frac{1}{n} \times T \times HLCF(H(h_i), A(a_i)) \le \frac{1}{n} \times T \times HLCF(H(h_i), B(b_i))$, which is $\boldsymbol{R}(T, HLCF(H(h_i), A(a_i))) \subseteq \boldsymbol{R}(T, HLCF(H(h_i), B(b_i)))$.

In summary, the $\boldsymbol{R}$ operator satisfies monotonicity.

Remark 5 The $\boldsymbol{R}$ operator here is neither a triangular norm (also called a t-paradigm) nor a fuzzy implication operator.

Definition 13 There are multiple rules which support the same conclusion, and when the multiple rules have parallel relationships, let

$$R_1 : A\{a_1, a_2, ..., a_m\} \to H\ ,\ (HLCF(H(h_i), A(a_i))$$
$$R_2 : B\{b_1, b_2, ..., b_m\} \to H\ ,\ (HLCF(H(h_i), B(b_i))$$

Where i represents the number of the hesitant fuzzy linguistic-valued credibility of the premises A, B and the conclusion H, $1 \le i \le m$. Define as:

$$HLCF_{a_i b_i}(h_i) = \begin{cases} LCF_S(h_i, a_i) \oplus^{\Delta} LCF_S(h_i, b_i) & , \quad LCF_S(h_i, a_i), LCF_S(h_i, b_i) \ge s_0 \\ LCF_S(h_i, a_i) + LCF_S(h_i, b_i) + LCF_S(h_i, a_i) \Theta LCF_S(h_i, b_i) & , \quad LCF_S(h_i, a_i), LCF_S(h_i, b_i) < s_0 \\ \dfrac{LCF_S(h_i, a_i) + LCF_S(h_i, b_i)}{s_n - \min\{|LCF_S(h_i, a_i)|, |LCF_S(h_i, b_i)|\}} & , \quad LCF_S(h_i, a_i) \times LCF_S(h_i, b_i) < s_0 \end{cases}$$

where

1) "Θ" is a bounded multiplication operator: $LCF_S(h_i, a_i) \Theta LCF_S(h_i, b_i) = \frac{1}{n} \times LCF_S(h_i, a_i) \times LCF_S(h_i, b_i)$;
2) "$\oplus^{\Delta}$" is a bounded addition operator: $LCF_S(h_i, a_i) \oplus^{\Delta} LCF_S(h_i, b_i) = LCF_S(h_i, a_i) + LCF_S(h_i, b_i) - \frac{1}{n} \times LCF_S(h_i, a_i) \times LCF_S(h_i, b_i)$.

Theorem 5 Let premise a_i and b_i be the parallel relation, and if they jointly derive the conclusion h_i, then the hesitant fuzzy linguistic-valued credibility $HLCF_{a_ib_i}(h_i)$ of h_i satisfies the boundedness, which range of values is $[s_{-n}, s_n]$.

Proof.

According to definition 13:

1) The first item of the equation (9) is $LCF_S(h_i,a_i)+LCF_S(h_i,b_i)-\frac{1}{n}\times LCF_S(h_i,a_i)\times LCF_S(h_i,b_i)$.

According to definition 4, we can know that $s_0 \le LCF_S(h_i,a_i) \le s_n$, $s_0 \le LCF_S(h_i,b_i) \le s_n$. So $s_0 \le (s_n - LCF_S(h_i,a_i)) \le s_n$, $s_0 \le (s_n - LCF_S(h_i,b_i)) \le s_n$. Then we can get $s_0 \le s_n - \frac{1}{n}\times(s_n - LCF_S(h_i,a_i))\times(s_n - LCF_S(h_i,b_i)) \le s_n$, i.e., $s_0 \le LCF_S(h_i,a_i)+LCF_S(h_i,b_i)-\frac{1}{n}\times LCF_S(h_i,a_i)\times LCF_S(h_i,b_i) \le s_n$. Then, $s_0 \le LCF_S(h_i,a_i) \oplus^{\Delta} LCF_S(h_i,b_i) \le s_n$.

Therefore, within the first item, $s_0 \le HLCF_{a_ib_i}(h_i) \le s_n$.

2) The second item of equation (9) is $LCF_S(h_i,a_i)+LCF_S(h_i,b_i)+\frac{1}{n}\times LCF_S(h_i,a_i)\times LCF_S(h_i,b_i)$.

As can be seen from definition 4, $s_{-n} \le LCF_S(h_i,a_i) < s_0$, $s_{-n} \le LCF_S(h_i,b_i) < s_0$. From that, we can obtain $s_{-n} < (s_{-n} - LCF_S(h_i,a_i)) \le s_0$, $s_{-n} < (s_{-n} - LCF_S(h_i,b_i)) \le s_0$. So $s_0 \le (s_{-n} - LCF_S(h_i,a_i))\times(s_{-n} - LCF_S(h_i,b_i)) < s_{n^2}$, and $s_{-n} \le s_{-n} - (-\frac{1}{n})\times(s_{-n} - LCF_S(h_i,a_i))\times(s_{-n} - LCF_S(h_i,b_i)) < s_0$. That is, $s_{-n} \le LCF_S(h_i,a_i)+LCF_S(h_i,b_i)+\frac{1}{n}\times LCF_S(h_i,a_i)\times LCF_S(h_i,b_i) < s_0$, e.t., $s_{-n} \le LCF_S(h_i,a_i)+LCF_S(h_i,b_i)+LCF_S(h_i,a_i)\Theta LCF_S(h_i,b_i) < s_0$.

Therefore, within the second item, $s_{-n} \le HLCF_{a_ib_i}(h_i) < s_0$.

3) As can be seen from definition 4, $s_0 \le LCF_S(h_i,a_i) \le s_n$, $s_{-n} \le LCF_S(h_i,b_i) < s_0$

The third item of the equation (9) is:

a) If $|LCF_S(h_i,a_i)| \ge |LCF_S(h_i,b_i)|$, then $s_0 \le LCF_S(h_i,a_i)+LCF_S(h_i,b_i) \le s_n$, and $\min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\} = |LCF_S(h_i,b_i)|$. Since $s_{-n} \le LCF_S(h_i,b_i) \le s_0$, so $s_0 \le |LCF_S(h_i,b_i)| \le s_n$.

Then we can get the $s_n - \min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\} = s_n - |LCF_S(h_i,b_i)|$, and $LCF_S(h_i,a_i)+LCF_S(h_i,b_i) = LCF_S(h_i,a_i) - |LCF_S(h_i,b_i)|$. Also because $LCF_S(h_i,a_i) \le s_n$, then $LCF_S(h_i,a_i) - |LCF_S(h_i,b_i)| \le s_n - |LCF_S(h_i,b_i)|$.

Therefore, we can get $s_0 \le \dfrac{LCF_S(h_i,a_i)+LCF_S(h_i,b_i)}{s_n - \min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\}} \le s_1$.

b) If $|LCF_S(h_i,a_i)|<|LCF_S(h_i,b_i)|$, then $s_{-n}\le LCF_S(h_i,a_i)+LCF_S(h_i,b_i)\le s_0$, and $\min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\}=|LCF_S(h_i,a_i)|$. When $s_0\le LCF_S(h_i,a_i)\le s_n$, we can get $s_0\le|LCF_S(h_i,a_i)|\le s_n$. Then $s_n-\min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\}=s_n-|LCF_S(h_i,a_i)|$, and $LCF_S(h_i,a_i)+LCF_S(h_i,b_i)=LCF_S(h_i,a_i)-|LCF_S(h_i,b_i)|$. And because $LCF_S(h_i,a_i)\le s_n$, the $LCF_S(h_i,a_i)-|LCF_S(h_i,b_i)|\le s_n-|LCF_S(h_i,b_i)|$ can be obtained.

Therefore, $s_{-1}\le\dfrac{LCF_S(h_i,a_i)+LCF_S(h_i,b_i)}{s_n-\min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\}}\le s_0$.

When $s_{-n}\le LCF_S(h_i,a_i)<s_0$, $s_0\le LCF_S(h_i,b_i)\le s_n$, the same can be proved that the third item of equation (9) is also $s_{-1}\le\dfrac{LCF_S(h_i,a_i)+LCF_S(h_i,b_i)}{s_n-\min\{|LCF_S(h_i,a_i)|,|LCF_S(h_i,b_i)|\}}\le s_1$.

To sum up, $HLCF_{a_ib_i}(h_i)$ satisfies boundedness and its range of values is $[s_{-n},s_n]$.

Definition 14 Let the expectation value of conclusion be H_{EXC}, and the conclusion H_A of scheme *A* and H_{EXC} are two hesitant fuzzy linguistic-valued credibility elements on the domain $HLCF=\{LCF_1,LCF_2,...,LCF_m\}$ respectively, then their closeness degree is defined as:

$$C(H_A,H_{EXC})=1/2\times[H_A\bullet H_{EXC}+(s_n-H_A\Theta H_{EXC})] \tag{10}$$

where,

$$H_A\bullet H_{EXC}=\underset{U}{\vee}(\mu_{H_A}(LCF_i)\wedge\mu_{H_{EXC}}(LCF_i)),$$

$$H_A\Theta H_{EXC}=\underset{U}{\wedge}(\mu_{H_A}(LCF_i)\vee\mu_{H_{EXC}}(LCF_i))$$

4.2 Reasoning Algorithm Based on Hesitant Fuzzy Linguistic-valued Credibility

Based on the hesitant fuzzy linguistic-valued credibility, the reasoning algorithm for the proposed method is as follows.

We can represent the reasoning rules R_g according to the HLCF knowledge representation method in definition 5.

Input: Evaluation table *EV*[m][n] and *EV*2[m][n], the number of experts is *q*, the reasoning rules R_g., and the expected value of conclusion H_{EXC}

Output: The best alternative X_{max}.

```
HLCFE(i, j, l){
   sum1= sum2= sum3= 0;
  for(k=0; k<q; k++){
   sum1+= EV[i][l].k* EV[j][l].k;
   sum2+= pow(EV[i][l].k, 2);
   sum3+= pow(EV[j][l].k, 2);
  }
return sum1 / sqrt(sum2) * sqrt(sum3);
} // Define the similarity function between alternatives
```

```
Bool com(double a, double b){
   return a>b;
}

int main ( ){
  for(i=1; i<=m; i++){
  for(j=1; j<=n; j++){
    p=0;
    for(k=1; k<=q; k++)
      if(EV[i][j].k!= Null){
        sum+= k;
          // sum represents the sum of the credibility evaluation values of the premise
with q experts
        p++;     // p represents the number of non-empty evaluation values
      }
      for(k=1; k<=q; k++){
                  if(EV[i][j].k==Null){
                  EV[i][j].k= sum/p;
           }
      sort(EV[i][j].1, EV[i][j].(k+1));
    }
  }
}
for(l=1; l<=n; l++)
  for(i=1; i<=m; i++)
    for(j=1; j<=m; j++)
    Rl[i][j]= HLCFE(i, j, l);  // Where Rl[i][j] represents the similarity matrix between
the alternatives under the lth attribute.
for(i=1; i<=m; i++){
  for (j=1; j<=m; j++){
    sum=0;
    for(l=1; l<=n; l++){
      sum+= Rl[i][j];
      RN[i][j]=  sum / n;
    }
  }
}

while(EV2[m][n] exists null values){
    max= -1; int p, q;
  for(i=1; i<=m; i++){
    for(j=1; j<=m; j++){
      if(max< RN[i][j] && i!=j){
        max= RN[i][j];
        p=i, q=j;
      }
    }
```

```
      }
   for(j=1; j<=n; j++){
    for(k=1; k<=q; k++){
        int v;
     v++;
        sum+= EV2[p][j].k;
        }
     if(EV2[p][j].k==Null && EV2[q][j].k==Null)
      EV2[p][j].k= sum/v*R_N[p][q];
    else if (EV2[p][j].k==Null){
     for(f=1; f<=q; f++){
           int v;
      v++;
           sum+= EV2[q][j].f;
           }
     EV2[p][j].k= sum/v*R_N[p][q];
     }
     else if (EV2[q][j].k==Null){
     for(g=1; g<=p; g++){
           int v;
      v++;
           sum+= EV2[p][j].g;
           }
     EV2[q][j].k = sum/v*R_N[p][q];
     }
  }
}
for (i=1;i<=m;i++){
   According to the given rules, the premises are aggregated to obtain Ω;
}
while( Ω !=NULL){
   if (r_i is a single rule supporting conclusion){// A' ∈ Ω
          F = 1/n*A'*HLCF(r_i);
             Put F in Ω_1;
          }
   else{
     if(r_1>= s_0 && r_2>= s_0){ // r_i、 r_i ∈r_g
          F = LCF_S(r_1)+LCF_S(r_2)-1/n*LCF_S(r_1)*LCF_S(r_2)
    }
     else if(r_1< s_0 && r_2< s_0){
        F = LCF_S(r_1)+LCF_S(r_2)+1/n*LCF_S(r_1)*LCF_S(r_2)
     }
     else{
        F = (LCF_S(r_1)+LCF_S(r_2))/(s_n-min(fbs(LCF_S(r_1)),fbs(LCF_S(r_2))))
     }
```

```
            Put F in Ω1;
      }
}
while( Ω2!=Null){ //Calculate the final conclusion H according to the actual situation
and store it in Ω2
      i=1;
      for(j=1;j<=q;j++){
      a[i]=1/2* (∨(LCF_S(H_i.j) ∧ LCF_S(H_Exc.j)) + (s_n − ∧(LCF_S(H_i.j) ∨ LCF_S(H_Exc.j))));
      }
      i++;
}
sort(a+1, a+i, com);
return a[1];
}
```

Restore the calculated subscripts to linguistic values.

5 An Example in Hesitant Fuzzy Linguistic-valued Credibility Uncertainty Reasoning

5.1 Practical Example Analysis

In order to avoid risks in advance, the Social Risk Analysis Bureau invites four experts to evaluate four cities: X_1, X_2, X_3, and X_4 from five aspects: Floods, Extreme temperatures, Seismic hazard, Population density, and Socio-economical instability. Furthermore, they determine where the most social risk is likely to occur. The risk analysis evaluation and reasoning rules given by experts are as follows. [see Tab. 1]

	Floods	Extreme temperatures	Seismic hazard	Population density	Socio-economical instability
X_1	$\{s_{-2},s_{-1},s_0,s_1\}$	$\{s_{-1},s_0,s_1,s_2\}$	$\{s_{-5},s_{-4},s_{-3},s_{-2}\}$	$\{s_2,s_3,s_4,s_5\}$	$\{s_{-4},s_{-3},s_{-2},\Delta\}$
X_2	$\{s_2,s_3,s_4,s_5\}$	$\{s_0,s_2,s_3,\Delta\}$	$\{s_3,s_4,s_5,\Delta\}$	$\{s_{-4},s_{-3},s_{-2},s_0\}$	$\{s_0,s_3,\Delta,\Delta\}$
X_3	$\{s_0,s_2,s_3,\Delta\}$	$\{s_3,s_4,s_5,\Delta\}$	$\{s_3,s_5,\Delta,\Delta\}$	$\{s_{-5},s_{-4},s_{-3},\Delta\}$	$\{s_2,s_3,s_4,s_5\}$
X_4	$\{s_2,s_4,s_5,\Delta\}$	$\{s_3,s_4,\Delta,\Delta\}$	$\{s_4,\Delta,\Delta,\Delta\}$	$\{s_3,s_4,s_5,\Delta\}$	$\{s_0,s_1,s_2,s_3\}$

Table 1: The risk analysis evaluation form for X_1, X_2, X_3, and X_4 four cities

The reasoning rules given by experts are as follows:

R_1: IF Floods AND Extreme temperatures OR Seismic hazard
THEN Environment risk $HLCF(R_1)=\{s_4,s_5,s_5,s_5\}$

R_2: IF Population density AND Socio-economical instability
THEN Social vulnerability $HLCF(R_2)=\{s_2,s_3,s_4,s_4\}$

R_3: IF Environment risk

THEN Social risk $HLCF(R_3) = \{s_3, s_4, s_4, s_5\}$

R4: IF Social vulnerability

THEN Social risk $HLCF(R_4) = \{s_3, s_3, s_4, s_5\}$

Step1: Let $A(X_i)$ indicate that X_i is prone to flooding; $B(X_i)$ expresses that X_i is prone to extreme temperatures; $C(X_i)$ represents that X_i is prone to seismic hazard; $D(X_i)$ denotes the population density of X_i; $E(X_i)$ shows that socio-economical in X_i is instability; $F(X_i)$ signifies that X_i is prone to environment risk; $G(X_i)$ indicates that X_i is prone to causing social vulnerability; $H(X_i)$ means that X_i is easy to have social risk; X_i is the city.

Symbolize the rules R_1~ R_4 as follows:

R_1: $\left(A(X_i) \wedge B(X_i) \vee C(X_i)\right) \to F(X_i)$

R_2: $\left(D(X_i) \wedge E(X_i)\right) \to G(X_i)$

R_3: $F(X_i) \to H(X_i)$

R_4: $G(X_i) \to H(X_i)$

Represent the reasoning rule R_1~ R_4 according to the *HLCF* knowledge representation method in definition 5.

R_1:

IF $A(X_i)$ $\left(HLCF(A) = \left\{LCF_S(a_{1i}), LCF_S(a_{2i}), LCF_S(a_{3i}), LCF_S(a_{4i})\right\}\right)$

AND $B(X_i)$ $\left(HLCF(B) = \left\{LCF_S(b_{1i}), LCF_S(b_{2i}), LCF_S(b_{3i}), LCF_S(b_{4i})\right\}\right)$

OR $C(X_i)$ $\left(HLCF(C) = \left\{LCF_S(c_{1i}), LCF_S(c_{2i}), LCF_S(c_{3i}), LCF_S(c_{4i})\right\}\right)$

THEN $F(X_i)$ $\left(HLCF(F) = \left\{LCF_S(f_{1i}), LCF_S(f_{2i}), LCF_S(f_{3i}), LCF_S(f_{4i})\right\}\right)$

$\left(HLCF(R_1) = \left\{s_4, s_5, s_5, s_5\right\}\right)$;

R_2:

IF $D(X_i)$ $\left(HLCF(D) = \left\{LCF_S(d_{1i}), LCF_S(d_{2i}), LCF_S(d_{3i}), LCF_S(d_{4i})\right\}\right)$

AND $E(X_i)$ $\left(HLCF(E) = \left\{LCF_S(e_{1i}), LCF_S(e_{2i}), LCF_S(e_{3i}), LCF_S(e_{4i})\right\}\right)$

THEN $G(X_i)$ $\left(HLCF(G) = \left\{LCF_S(g_{1i}), LCF_S(g_{2i}), LCF_S(g_{3i}), LCF_S(g_{4i})\right\}\right)$

$\left(HLCF(R_2) = \{s_2, s_3, s_4, s_4\}\right)$;

R_3:

IF $F(X_i)$ $\left(HLCF(F) = \left\{LCF_S(f_{1i}), LCF_S(f_{2i}), LCF_S(f_{3i}), LCF_S(f_{4i})\right\}\right)$

THEN $H(X_i)$ $\left(HLCF(H) = \left\{LCF_S(h_{1i}), LCF_S(h_{2i}), LCF_S(h_{3i}), LCF_S(h_{4i})\right\}\right)$

$\left(HLCF(R_3) = \{s_3, s_4, s_4, s_5\}\right)$;

R_4:

IF $G(X_i)$ $\left(HLCF(G) = \left\{LCF_S(g_{1i}), LCF_S(g_{2i}), LCF_S(g_{3i}), LCF_S(g_{4i})\right\}\right)$

THEN $H(X_i)$ $\left(HLCF(H) = \left\{LCF_S(h_{1i}), LCF_S(h_{2i}), LCF_S(h_{3i}), LCF_S(h_{4i})\right\}\right)$

$\left(HLCF(R_4)=\{s_3,s_3,s_4,s_5\}\right)$.

Establish a reasoning model, as shown in [Fig. 1].

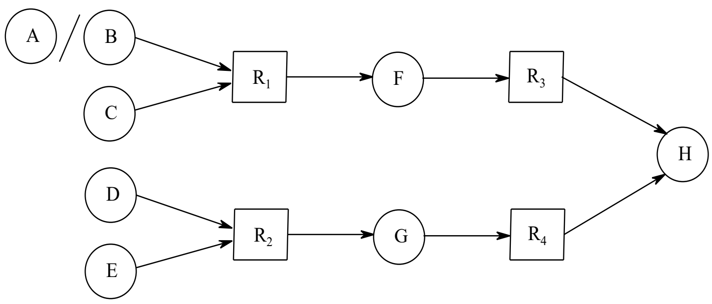

Figure 1: Diagram of uncertainty reasoning based on HLCF

Step2: First, we initially complete the table1 by averaging. We can obtain a new risk analysis evaluation form for X_1, X_2, X_3, and X_4 four cities. ([see Tab. 2], the added elements are boldly marked)

	Floods	Extreme temperatures	Seismic hazard	Population density	Socio-economical instability
X_1	$\{s_{-2},s_{-1},s_0,s_1\}$	$\{s_{-1},s_0,s_1,s_2\}$	$\{s_{-5},s_{-4},s_{-3},s_{-2}\}$	$\{s_2,s_3,s_4,s_5\}$	$\{s_{-4},s_{-3},\boldsymbol{s_{-3}},s_{-2}$
X_2	$\{s_2,s_3,s_4,s_5\}$	$\{s_0,s_2,\boldsymbol{s_2},s_3\}$	$\{s_3,s_4,\boldsymbol{s_4},s_5\}$	$\{s_{-4},s_{-3},s_{-2},s_0\}$	$\{s_0,\boldsymbol{s_2},\boldsymbol{s_2},s_3\}$
X_3	$\{s_0,s_2,\boldsymbol{s_2},s_3\}$	$\{s_3,s_4,\boldsymbol{s_4},s_5\}$	$\{s_3,\boldsymbol{s_4},\boldsymbol{s_4},s_5\}$	$\{s_{-5},s_{-4},\boldsymbol{s_{-4}},s_{-3}\}$	$\{s_2,s_3,s_4,s_5\}$
X_4	$\{s_2,s_4,\boldsymbol{s_4},s_5\}$	$\{s_3,s_4,\boldsymbol{s_4},\boldsymbol{s_4}\}$	$\{s_4,\boldsymbol{s_4},\boldsymbol{s_4},\boldsymbol{s_4}\}$	$\{s_3,s_4,\boldsymbol{s_4},s_5\}$	$\{s_0,s_1,s_2,s_3\}$

Table 2: The initial completion form for risk analysis evaluation

Step2.1 Let $t=1$and calculate the similarity degrees $sim_{c_r}\left(X_i,X_j\right)$of any two alternatives $X_i$and $X_j$of attribute $c_r$to obtain the similarity matrix $R_{c_r}^{(1)}=\left(x_{ij}\right)_{4\times4}$, where $1\le i,j\le 4$, $1\le r\le 5$. Then we get the similarity aggregation matrix $R_{\aleph(c_r)}^{(1)}=\left(y_{ij}\right)_{4\times4}$of c_r.

$$R_{Flo}^{(1)}=\begin{pmatrix} s_1 & s_{-0.111} & s_{0.099} & s_{-0.157} \\ s_{-0.111} & s_1 & s_{0.957} & s_{0.993} \\ s_{0.099} & s_{0.957} & s_1 & s_{0.963} \\ s_{-0.157} & s_{0.993} & s_{0.963} & s_1 \end{pmatrix} \qquad R_{Ext}^{(1)}=\begin{pmatrix} s_1 & s_{0.792} & s_{0.553} & s_{0.487} \\ s_{0.792} & s_1 & s_{0.925} & s_{0.899} \\ s_{0.553} & s_{0.925} & s_1 & s_{0.995} \\ s_{0.487} & s_{0.899} & s_{0.995} & s_1 \end{pmatrix}$$

$$R_{Sei}^{(1)} = \begin{pmatrix} s_1 & s_{-0.888} & s_{-0.888} & s_{-0.953} \\ s_{-0.888} & s_1 & s_1 & s_{0.985} \\ s_{-0.888} & s_1 & s_1 & s_{0.985} \\ s_{-0.953} & s_{0.985} & s_{0.985} & s_1 \end{pmatrix} \quad R_{Pop}^{(1)} = \begin{pmatrix} s_1 & s_{-0.632} & s_{-0.888} & s_{0.988} \\ s_{-0.632} & s_1 & s_{0.914} & s_{-0.731} \\ s_{-0.888} & s_{0.914} & s_1 & s_{-0.939} \\ s_{0.988} & s_{-0.731} & s_{-0.939} & s_1 \end{pmatrix}$$

$$R_{Soc}^{(1)} = \begin{pmatrix} s_1 & s_{-0.708} & s_{-0.861} & s_{-0.650} \\ s_{-0.708} & s_1 & s_{0.957} & s_{0.972} \\ s_{-0.861} & s_{0.957} & s_1 & s_{0.946} \\ s_{-0.650} & s_{0.972} & s_{0.946} & s_1 \end{pmatrix} \quad R_{\aleph}^{(1)} = \begin{pmatrix} s_1 & s_{-0.309} & s_{-0.397} & s_{-0.057} \\ s_{-0.309} & s_1 & s_{0.951} & s_{0.624} \\ s_{-0.397} & s_{0.951} & s_1 & s_{0.590} \\ s_{-0.057} & s_{0.624} & s_{0.590} & s_1 \end{pmatrix}$$

Step2.2 We can see $y_{23} = y_{32}$ is the largest from $R_{\aleph}^{(1)}$ easily, that is, the similarity of X_2 and X_3 is the largest. Arranging $EV^{(0)}$, we can get $EV^{(1)}$. ([see Tab. 3], the added elements are boldly marked)

	Floods	Extreme temperatures	Seismic hazard	Population density	Socio-economical instability
X_1	$\{s_{-2}, s_{-1}, s_0, s_1\}$	$\{s_{-1}, s_0, s_1, s_2\}$	$\{s_{-5}, s_{-4}, s_{-3}, s_{-2}\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, \Delta\}$
X_2	$\{s_2, s_3, s_4, s_5\}$	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{s_3, \mathbf{s_{3.8}}, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, s_0\}$	$\{s_0, s_3, \mathbf{s_{3.3}}, \mathbf{s_{3.3}}\}$
X_3	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{\mathbf{s_{1.6}}, s_3, s_4, s_5\}$	$\{s_3, \mathbf{s_{3.8}}, \mathbf{s_{3.8}}, s_5\}$	$\{s_{-5}, s_{-4}, s_{-3}, \mathbf{s_{-2.1}}\}$	$\{s_2, s_3, s_4, s_5\}$
X_4	$\{s_2, s_4, s_5, \Delta\}$	$\{s_3, s_4, \Delta, \Delta\}$	$\{s_4, \Delta, \Delta, \Delta\}$	$\{s_3, s_4, s_5, \Delta\}$	$\{s_0, s_1, s_2, s_3\}$

Table 3: The first fine completion form for risk analysis evaluation

Thus, we get $EV^1 \neq EV$ from Table 3. Then, return to **Step2.1**.

Let t=2, due to the similarity of X_2 and X_3 are complete, overlook $sim_{c_r}(X_2, X_3)$. We can obtain $y_{24} = y_{42}$ is the largest from $R_{\aleph}^{(1)}$ easily, that is, the similarity of X_2 and X_4 is the largest. Arranging matrix $EV^{(1)}$, we can get $EV^{(2)}$. ([see Tab. 4], the added elements are boldly marked)

	Floods	Extreme temperatures	Seismic hazard	Population density	Socio-economical instability
X_1	$\{s_{-2}, s_{-1}, s_0, s_1\}$	$\{s_{-1}, s_0, s_1, s_2\}$	$\{s_{-5}, s_{-4}, s_{-3}, s_{-2}\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, \Delta\}$
X_2	$\{s_2, s_3, s_4, s_5\}$	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{s_3, \mathbf{s_{3.8}}, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, s_0\}$	$\{s_0, s_3, \mathbf{s_{3.3}}, \mathbf{s_{3.3}}\}$
X_3	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{\mathbf{s_{1.6}}, s_3, s_4, s_5\}$	$\{s_3, \mathbf{s_{3.8}}, \mathbf{s_{3.8}}, s_5\}$	$\{s_{-5}, s_{-4}, s_{-3}, \mathbf{s_{-2.1}}\}$	$\{s_2, s_3, s_4, s_5\}$
X_4	$\{s_2, \mathbf{s_{2.2}}, s_4, s_5\}$	$\{\mathbf{s_{1.3}}, \mathbf{s_{1.3}}, s_3, s_4\}$	$\{\mathbf{s_{2.5}}, \mathbf{s_{2.5}}, \mathbf{s_{2.5}}, s_4\}$	$\{\mathbf{s_{-1.4}}, s_3, s_4, s_5\}$	$\{s_0, s_1, s_2, s_3\}$

Table 4: The second fine completion form for risk analysis evaluation

Thus, we can get $EV^2 \neq EV$, return to **Step2.1**.

Let t=3, due to the similarity of X_2, X_3 and X_4 are complete, overlook $sim_{c_r}\left(X_2, X_3\right)$, $sim_{c_r}\left(X_2, X_4\right)$ and $sim_{c_r}\left(X_3, X_4\right)$. We can see $y_{14} = y_{41}$ is the largest from $R_{\aleph}^{(1)}$ easily, that is, the similarity of X_1 and X_4 is the largest. Arranging matrix $EV^{(2)}$, we can get $EV^{(3)}$. ([see Tab. 5], the added elements are boldly marked)

	Floods	Extreme temperatures	Seismic hazard	Population density	Socio-economical instability
X_1	$\{s_{-2}, s_{-1}, s_0, s_1\}$	$\{s_{-1}, s_0, s_1, s_2\}$	$\{s_{-5}, s_{-4}, s_{-3}, s_{-2}\}$	$\{s_2, s_3, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, \mathbf{s_{-0.1}}\}$
X_2	$\{s_2, s_3, s_4, s_5\}$	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{s_3, \mathbf{s_{3.8}}, s_4, s_5\}$	$\{s_{-4}, s_{-3}, s_{-2}, s_0\}$	$\{s_0, s_3, \mathbf{s_{3.3}}, \mathbf{s_{3.3}}\}$
X_3	$\{s_0, s_2, s_3, \mathbf{s_{3.3}}\}$	$\{\mathbf{s_{1.6}}, s_3, s_4, s_5\}$	$\{s_3, \mathbf{s_{3.8}}, \mathbf{s_{3.8}}, s_5\}$	$\{s_{-5}, s_{-4}, s_{-3}, \mathbf{s_{-2.1}}\}$	$\{s_2, s_3, s_4, s_5\}$
X_4	$\{s_2, \mathbf{s_{2.2}}, s_4, s_5\}$	$\{\mathbf{s_{1.3}}, \mathbf{s_{1.3}}, s_3, s_4\}$	$\{\mathbf{s_{2.5}}, \mathbf{s_{2.5}}, \mathbf{s_{2.5}}, s_4\}$	$\{\mathbf{s_{-1.4}}, s_3, s_4, s_5\}$	$\{s_0, s_1, s_2, s_3\}$

Table 5: The third fine completion form for risk analysis evaluation

Thus, we can get $EV^3 = EV$, jump to **Step 3**.

Step3: Calculate the HLCF of social risk in each city when the above conditions are established, so as to determine which city is most prone to social risk.

For the city X_1, as can be seen from Table 5, in R_1 we can get

$A(X_1)$ AND $B(X_1)$ OR $C(X_1)$ (denoted by $A(X_1)'$)

$$= A(X_1) \wedge B(X_1) \vee C(X_1)$$

$$= \left\{s_{-2}, s_{-1}, s_0, s_1\right\}$$

R_1 is a single rule supporting the conclusion of $F\left(X_1\right)$, according to equation (8)

$$HLCF\left(F\left(X_1\right)\right) = \frac{1}{n} \times HLCF\left(A\left(X_1\right)'\right) \times HLCF(R_1) = \{s_{-1.6}, s_{-1}, s_0, s_1\}$$

Similarly, as can be seen from Table 5, in R_2 we can get

$D\left(X_1\right)$ AND $E\left(X_1\right)$ (denoted by $D\left(X_1\right)'$)

$$= D\left(X_1\right) \wedge E\left(X_1\right)$$

$$= \left\{s_{-4}, s_{-3}, s_{-2}, s_{-0.1}\right\}$$

R_2 is a single rule supporting the conclusion of $G\left(X_1\right)$, according to equation (8)

$$HLCF\left(G\left(X_1\right)\right) = \frac{1}{n} \times HLCF\left(D\left(X_1\right)'\right) \times HLCF(R_2) = \left\{s_{-1.6}, s_{-1.8}, s_{-1.6}, s_{-0.08}\right\}$$

The $HLCF\left(F\left(X_1\right)\right)$ is $\left\{s_{-1.6}, s_{-1}, s_0, s_1\right\}$, and R_3 is a single rule supporting the conclusion of $H_1\left(X_1\right)$, according to equation (8)

$$HLCF\left(H_1\left(X_1\right)\right)=\frac{1}{n}\times HLCF\left(F\left(X_1\right)\right)\times HLCF(R_3)=\left\{s_{-0.96},s_{-0.8},s_0,s_1\right\}$$

The $HLCF\left(G\left(X_1\right)\right)$is $\left\{s_{-1.6},s_{-1.8},s_{-1.6},s_{-0.08}\right\}$, and R_4 is a single rule supporting the conclusion of $H_2\left(X_1\right)$, according to equation (8)

$$HLCF\left(H_2\left(X_1\right)\right)=\frac{1}{n}\times HLCF\left(G\left(X_1\right)\right)\times HLCF(R_4)=\left\{s_{-0.96},s_{-1.08},s_{-1.28},s_{-0.08}\right\}$$

Since R_3 and R_4 are parallel rules and support the same conclusion $H\left(X_1\right)$, jump to **Step4**.

Step4: Since the rules R_3 and R_4 are parallel relationship, according to Equation (9), the hesitant fuzzy linguistic-valued credibility $HLCF\left(H\left(X_1\right)\right)$of the conclusion $H\left(X_1\right)$is obtained.

Therefore, $HLCF\left(H\left(X_1\right)\right)=\left\{s_{-1.736},s_{-1.707},s_{-0.256},s_{0.187}\right\}$.

In the same way, the social risk situation of X_2~ X_4 cities can be calculated

$HLCF\left(H\left(X_2\right)\right)=\left\{s_{0.119},s_{0.500},s_{0.516},s_5\right\}$;

$HLCF\left(H\left(X_3\right)\right)=\left\{s_{0.063},s_{0.449},s_{0.364},s_1\right\}$;

$HLCF\left(H\left(X_4\right)\right)=\left\{s_{0.185},s_{2.216},s_{3..066},s_{4.480}\right\}$.

Step5: Let H_{EXC}be $\left\{s_4,s_4,s_5,s_5\right\}$. Calculate the closeness degrees between the conclusion $H(X_i)$of each alternative X_i（i=1,2,3,4）and the expectation value H_{EXC}.

For the city X_1, according to equation (10), we can obtain the $C\left(H(X_1),H_{EXC}\right)$as follows

$$H_{X_1}\bullet H_{EXC}=\left(s_4\wedge s_{-1.736}\right)\vee\left(s_4\wedge s_{-1.707}\right)\vee\left(s_5\wedge s_{-0.256}\right)\vee\left(s_5\wedge s_{0.187}\right)=s_{0.187}$$

$$H_{X_1}\Theta H_{EXC}=\left(s_4\vee s_{-1.736}\right)\wedge\left(s_4\vee s_{-1.707}\right)\wedge\left(s_5\vee s_{-0.256}\right)\wedge\left(s_5\vee s_{0.187}\right)=s_4$$

$$C\left(H(X_1),H_{EXC}\right)=\frac{1}{2}\times\left[s_{0.187}+(s_5-s_4)\right]=s_{0.594}$$

In the same way, the closeness degrees between conclusion $H(X_2)$~ $H(X_4)$and H_{EXC} are as follows

$$C\left(H(X_2),H_{EXC}\right)=\frac{1}{2}\times\left[s_5+(s_5-s_4)\right]=s_3$$

$$C\left(H(X_3),H_{EXC}\right)=\frac{1}{2}\times\left[s_1+(s_5-s_4)\right]=s_1$$

$$C\big(H(X_4),H_{EXC}\big)=\frac{1}{2}\times\big[s_{4.48}+(s_5-s_4)\big]=s_{2.74}$$

Step6: Sort closeness degrees, we can get $s_3 > s_{2.74} > s_1 > s_{0.594}$. So the city X_2 is the most prone to social risks. We should take appropriate precautions against the city X_2 in advance to avoid risk.

The example illustrates the effectiveness and feasibility of knowledge representation and knowledge reasoning based on hesitant fuzzy linguistic-valued credibility. Due to the fact that the operations used in the reasoning process are based on the hesitant fuzzy linguistic term sets, which are again based on logical axioms and express hesitant preferences of experts better. This justifies the rationality of the proposed uncertainty knowledge reasoning for decision making.

5.2 Comparative Analysis

1) [Gao et al. 2019] combined the weights of the clinical signs and the promoted confidence factors (PCF) to express uncertainty information and rules pertaining to clinical signs, and developed a reasoning method for equine disease diagnosis in [Gao et al. 2019]. The [Gao et al. 2019] iterates each diagnosis result by raising the frequency fi so that obtained credibility of the premises is more accurate. However, the reasoning rules of this method are single and cannot meet the needs of the reasoning process well. Therefore, this paper proposes the reasoning method of single rule and multiple rules with parallel relationship, which both considers the impact of the credibility of premises and rules on the reasoning results, making the results more accurate.

2) The [Gao et al. 2019] uses fuzzy membership function to represent premises and rules, and the knowledge representation was expressed using production rules. This paper proposes the hesitant fuzzy linguistic-valued credibility to represent premises and rules, which can express the hesitant preference of experts in the process of evaluating premise better. It can minimize the inaccuracy of credibility assessments and avoid missing information.

3) Numerical values are used to describe the credibility of the premises and rules in the [Gao et al. 2019]. However, uncertainty is produced by the vagueness of meanings whose nature is qualitative rather than quantitative in many cases. People are more accustomed to express with linguistic values rather than numerical values when conducting evaluation reasoning. Therefore, this paper uses the linguistic-valued credibility to express the hesitant preference of experts, which is closer to the human thinking.

6 Conclusion and Outlook

Credibility reasoning is considered as a powerful tool to express uncertain information in the process of reasoning and decision making. Therefore, this paper combines the hesitant fuzzy linguistic term set with the credibility method to define the hesitant fuzzy linguistic-valued credibility, which uses a set of hesitant linguistic values to represent the credibility of premise and rules. It can effectively solve the problem of inaccurate estimation of credibility. Then, in order to solve the problem of incomplete information, this paper proposes an information complete algorithm for hesitant fuzzy linguistic-valued credibility based on the maximum similarity. Furthermore, the paper constructs a knowledge representation method for hesitant fuzzy linguistic-valued credibility. And the reasoning rules of hesitant fuzzy linguistic-valued credibility are proposed, which mainly include two kinds of rules: single rule supporting the conclusion, and parallel relationship of multiple rules. Finally, a practical example involving social risk analysis is used to verify the efficiency and applicability of the proposed approach.

The rational decision making methods with the theoretical foundations to address the accuracy, reliability and precision of decision making have been hot research topic owing to their importance and effectiveness. To a certain extent, this paper studies the reasonable decision making method based on hesitant fuzzy linguistic-valued credibility reasoning. For future work, we can further discuss other reasoning methods based on hesitant fuzzy linguistic values and use hesitant fuzzy linguistic values to deal with the reasoning problem in the concept lattice, which will be a more meaningful study.

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